

Capacitance, C



A capacitor is two metal plates with a space between them that is non-conducting.

When a capacitor is connected to a voltage source, charge builds up. The amount of charge is determined by the size of the plates, the distance between the plates and the material in the space between them.

$$C = \epsilon A/d$$

C is capacitance in Farads, $F = C/V$

A is the area of the smallest of the metal plates in m^2 (not added up)

d is the distance between the plates in m

ϵ is permittivity of the material between the plates.

for a vacuum, $\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

The charge build up on the plates is related to the voltage

$$V = \text{energy}/q$$

$$W = Fd$$

$$E = V/d$$

$$C = q/V$$

eg. A capacitor is built using a 2.0 cm² sheet of copper 3.0 mm apart. Determine
a) the capacitance if the spacing is air

$$C = \epsilon A/d$$

$$= 8.85 \times 10^{-12} \times (0.0002) / 0.003 = 0.0$$

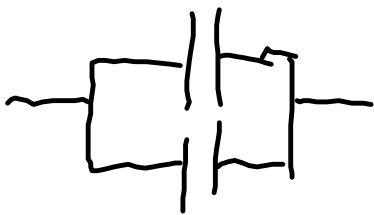
$$5.9 \times 10^{-13} \text{ F} = 0.59 \text{ pF}$$

$$\text{C}^2 \text{N}^{-1} \text{m}^{-2} \times \text{m}^2/\text{m} \quad \text{C}^2/\text{Nm} = \text{C}^2/\text{J} = \text{C}/\text{V}$$

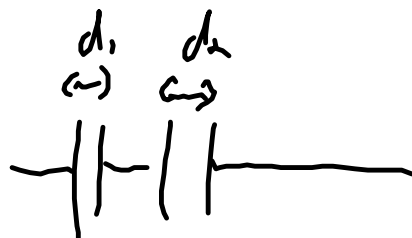
a) the charge on the capacitor if it is connected to a 6.0V battery

$$q = CV = 0.59 \times 6 = 3.54 \text{ pC}$$

a) If you put 2 capacitors in series or in parallel, how would that influence the total capacitance?



Hecht p592-



$$C = \frac{\epsilon A}{d}$$

$$C = C_1 + C_2 \text{ in parallel}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \text{ Series}$$

$$R = \frac{\rho L}{A}$$

H

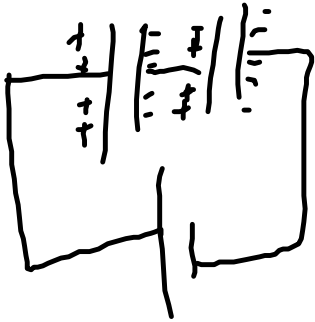
- if they are in parallel $C = C_1 + C_2$
 $C = 2C_1$
 $C = 1.18 \text{ pF}$

- if they are in series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_T} = \frac{1}{0.59 \text{ pF}} + \frac{1}{0.59 \text{ pF}}$$

$$C_T = 0.295 \text{ pF}$$



$$q = q_0 e^{-\frac{t}{\tau}}$$

$$\tau = R C$$

↑
resistance
of the
circuit

t is the time for the charge to build up or discharge on/off the capacitor.

eg. you connect a 0.59 pF capacitor to a 4.0Ω resistor and a 6.0 V battery. Determine
 a) the maximum charge on the capacitor?

$$C=q/V \quad q=CV = 0.59 \times 6 = 3.54 \text{ pC}$$

a) the time for the capacitor to be halfway charged.

$$q/q_0 = 1/2 = e^{-t/\tau}$$

q_0 is maximum charge

q is the charge at time t

$$\tau = RC = 4 \times 0.59 = 2.36 \times 10^{-12} \text{ } \Omega\text{F}$$

$$\ln(1/2) = -0.693147180559945 = -t/2.3 \times 10^{-12}$$

$$t = 0.693147180559945 \times 2.3 =$$

$$1.594238515287874$$

$$1.59 \text{ ps}$$

a) what is the charge at two half-lives?

$$t = 2 \times 1.59 = 3.18$$

$$q = q_0 e^{-t/\tau}$$

$$= 3.54 \text{ pC } e^{-3.18 \text{ ps} / 2.36 \Omega\text{pF}}$$

$$3.54 \times 2.718^{(-3.18/2.36)} = 0.9202$$

$$0.92 \text{ pC}$$

a) how much energy is stored on the capacitor?

$$V = \text{energy/charge}$$

$$\text{energy} = Vq$$

$$C = q/V$$

$$q = CV$$

$$\text{energy} = V^2 C \text{ wrong!}$$

the charge builds up over time

energy = integral of the buildup

energy = $\frac{1}{2}CV^2$ (the field and therefore the potential builds up linearly with charge, so the area under the V-q graph is the area of a triangle = energy)

energy stored in a capacitor = $\frac{1}{2} CV^2$

= $0.5 \times 0.59 \times 6 \times 6 = 10.62 = 11 \text{ pJ}$

Hecht p610-614
49, 67, 81, 87, 97, 104

Physics Olympics, Saturday March 10th
contests: AAPT April 6th block 2-1 (\$5)
CAP April 11th 9-12 (\$2) (blocks 1-4, 1-3)